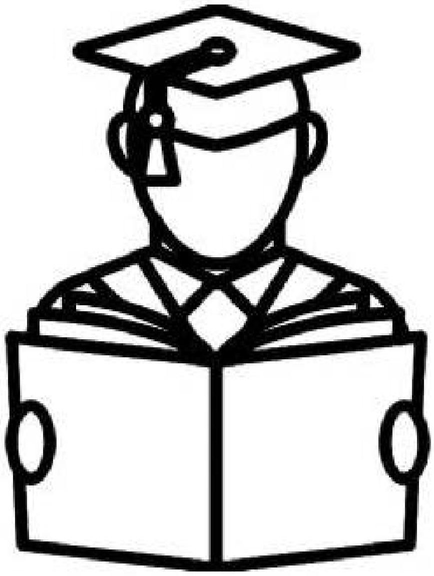
चौधरी PHOTOSTAT

"I don't love studying. I hate studying. I like learning. Learning is beautiful."



"An investment in knowledge pays the best interest."

Hi, My Name is

Mathematical Science for CSIR NET

Gurukulam(Guru)

Chapter-01 Lin

$(F, \oplus, \odot)$ is field.

$\oplus : F \times F \longrightarrow F$

$\odot : F \times F \longrightarrow F$

$+ : V \times V \longrightarrow V$

$\cdot : F \times V \longrightarrow V$

Field

Let F be a non empty set and $+, \cdot$ are two binary operations on F

$+ : F \times F \to F$ are functions

$\cdot : F \times F \to F$

Then $(F, +, \cdot)$ is called field if

(i) $(F, +)$ is abelian group.

(ii) $(F - \{0\} = F^*, \cdot)$ is abelian group.

(iii) $\cdot$ is distributive over '+'.

example :- 1) $(\mathbb{C}, +, \cdot)$

2) $(\mathbb{R}, +, \cdot)$

3) $(\mathbb{Q}, +, \cdot)$

4) $\mathbb{Q}(\sqrt{P}) = \{a + b\sqrt{P} \mid a, b \in \mathbb{Q}\}$; P is Prime no.

5) $(\mathbb{Z}_p, +_p, \times_p)$; P is Prime

6) $(\mathbb{R}^+, *, \#)$; whether this structure is field or no

$a * b = a \cdot b$

$a \# b = a^{\log b}$

Now, (i) $(\mathbb{R}^+, *)$ is abelian group.

(ii) $(\mathbb{R}^+ - \{1\}, \#)$ is abelian group.

(iii) $\#$ is distributive over $*$.

$a \# (b * c) = (a \# b) * (b \# c)$

$a^{\log(b \cdot c)} = a^{\log b} \cdot a^{\log c}$

$a * (b \# c) = (a \# c) * (b \# c)$

$u = (ab)^{\log c}$, $v = a^{\log c} \cdot b^{\log c}$

$\log u = \log v$

$\Rightarrow u = v$

7) $(\mathbb{R}^+, *, \#)$

$\quad \cdot \quad a^{\log b}$

$(F, +, \cdot)$

<u>Note</u>:- 1) for every natural no. $n \in \mathbb{N}$ $\exists$ a field F such that $|F| = p^n$

2) If F be a finite field Then $|F| = p^k$ for some p-prime, $K \in \mathbb{N}$.

i.e. Cardinality of finite field never divisible by any two Prime nos.

3) $W = \langle \alpha_1 x_1 + \alpha_2 x_2 \rangle$

$\alpha_i \in F$; $|F| = p^K$

$\Rightarrow |W| = p^K \cdot p^K$

$= p^{2K}$

$\Rightarrow |W| = p^{2K}$

mycompanion PLUS

Vector Space

Let $V \neq \phi$ and $(F, +, \cdot)$ be field.
Define $*: V \times V \to V$ (function) (Internal composition)
$\#: F \times V \to V$ (function) (External composition)

i.e. Addition of vector $+ : V \times V \to V$
Scalar Multiplication to the vectors $\cdot : F \times V \to V$

[~~It is~~ $+, \cdot$ are just symbols and not addition & multiplication]

Then V is called vector space over field F w.r.t $*$, $\#$ if

(i) $(V, *)$ is abelian group.

(ii) $\forall \alpha, \beta \in F$, $u, v \in V$

$(\alpha \cdot \beta) \# u = (\alpha \# u) * (\beta \# u)$ (Right distributive)

$\#$ is distributive over $*$.

(iii) $\alpha \# (u * v) = (\alpha \# u) \cdot (\alpha \# v)$ (Left distributive)

(iv) $(\alpha \cdot \beta) \# u = \alpha \# (\beta \# u)$ (Associativity)

(v) $1 \# u$
unity of field
(Identity element of M in $(F^*, \cdot)$

<u>example</u>:- $V = \{ f_a : \mathbb{R} \to \mathbb{R} \mid f_a(x) = a + x \;;\; a \in \mathbb{R} \}$

$(\mathbb{R}, +, \cdot)$

$* : V \times V \to V$

$f_a * f_b = f_a \circ f_b = f_{a+b}$

$\# : F \times V \to V$

$\alpha \# f_a = f_{(\alpha \cdot a)}$

$$\begin{aligned} f_a \circ f_b(x) &= f_a(b+x) \\ &= a+b+x \\ &= f_{a+b}(x) \end{aligned}$$

$\Rightarrow f_a \circ f_b = f_{a+b}$

Now, $*$ is B.O

$*$ is Associative

$f_0(x) = x = I(x)$; $I(x)$ is Identity.

$f_a^{-1} = f_{-a}$

$*$ is commutative

$\therefore (\mathbb{R}, +)$ is abelian.

$\rightarrow \forall \alpha, \beta \in \mathbb{R}$; $f_a, f_b \in V$

$\rightarrow \underline{(\alpha+\beta) \# f_a = f_{(\alpha+\beta)a}}$

$$\begin{aligned} (\alpha \# f_a) * (\beta \# f_a) &= f_{\alpha a} * f_{\beta a} \\ &= f_{\alpha a + \beta a} \end{aligned}$$

Thus $(\alpha+\beta) \# f_a = (\alpha \# f_a) * (\beta \# f_a)$

$f_{(\alpha+\beta)a} = f_{\alpha a} * f_{\beta a}$

or $f_{(\alpha a + \beta a)}$

companion

Group Theory

→ Sets, functions, relations
→ Group, subgroup, order of elements
→ Cyclic group, group of order 4, 6, P, q, P^2, $(C^*, \cdot)$
→ Group of Bijections, Group of symmetries (Imp-Gate)
→ Class eqⁿ, normal subgroup
→ Homo on G, Quotient group
→ Sylow's Theorem

Sets :- Collection of well defined distinct objects.
Subsets :- Let A & B be any sets
$A \subseteq B$ iff $x \in A \Rightarrow x \in B$
$\phi \subseteq A$ $\forall A$
Proof :- Let $\phi \not\subseteq A$
$\exists\, x \in \phi$ & $x \notin A$
Contradicts
$\Rightarrow \phi \subseteq A$

$|A| = n$
$\Rightarrow$ No of subsets of $A = 2^n$
Proof :- No. of subsets $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$

Binomial Theorem

$(1+x)^n = {}^nC_0 + x\,{}^nC_1 + x^2\,{}^nC_2 + \dots + x^n\,{}^nC_n$
Put $x=1$
${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

DATE

Power Set $P(A)$

If A is any set
$P(A) = \{X \mid X \subseteq A\}$
$\rightarrow P(A) \neq \phi$

Functions

Let $A = \phi$, $B = \phi$
Then $f: A \rightarrow B$ is called function if $\forall x \in A \; \exists \; y(\text{unique}) \in B$
such that $f(x) = y$

Geometric Defination

Let $A, B \subseteq R$, we represent A on X-axis, B on Y-axis, Then a mapping $f: A \rightarrow B$ is called a function if every line Passing through A and Parallel to B intersect the curve $y = f(x)$ exactly once

one-one function

If $f: A \rightarrow B$ & f is 1-1.
If $\forall \; x_1 \neq x_2$; $x_1, x_2 \in A$
$\Rightarrow f(x_1) \neq f(x_2)$
or If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

onto function

If $f: A \rightarrow B$
Then f is onto if $\forall \; y \in B \; \exists \; x \in A$. Such that $y = f(x)$
$f(x_1) = y_1$ and $f(x_2) = y_2$; $x_1, x_2 \in A$
$\Rightarrow x_1 \neq x_2$

Que:- No. of 1-1 and onto function from A to B
$|A| = n$, $|B| = m$.

Que:- Let $|A| = 2n+1$, $n \geq 2$

Find no. of subsets of A with more than 'n' elements.

(a) 2^n

(b) $2^{2n}-1$

(c) 2^{2n}

(d) 2^{2n-1}

Sol:- No. of required subsets $= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1}$

$= \alpha$

Total subsets of A $= {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + \ldots + {}^{2n+1}C_{2n+1}$

${}^nC_r = {}^nC_{n-r}$

$\Rightarrow \alpha + \alpha = 2^{2n+1}$

$\Rightarrow 2\alpha = 2^{2n+1}$

$\Rightarrow \alpha = 2^{2n}$

(c)

Que:- $|A| = 100$

TIFR-16 $S = \{X \mid X \subseteq A\}$

Find: maxi $|S|$ $\forall X, Y \in S \Rightarrow X \cap Y \neq \phi$

(a) 2^{99}

(b) $2^{100}-1$

(c) $2^{99}-1$

(d) none

Sol:- $a \in A$

s.t. $a \in X \cap Y$ $\forall X, Y \in S$

Define $B = A - \{a\}$

$|B| = 99$

Subsets $= 2^{99}$

$X \subseteq B \Rightarrow X_1 = X \cup \{a\}$

$S = \{X_1 \mid X_1 \subseteq A\}$

Lion (a)

DATE

Que:- $|A|=2n$; A has successive $2n$ natural no. If for any $B \subseteq A$ $\exists\, a,b \in B$ s.t $\gcd(a,b)=1$. Then least no. of elements in B

(a) 2

(b) $n-1$

(c) n

(d) $n+1$

Sol:- $A=\{1,2,3,\ldots,2n\}$

$|B|=n$

If $B=\{2,4,6,\ldots\}$ Then $\nexists$ any consecutive a,b term s.t $\gcd(a,b)=1$

If $B=\{1,3,5,\ldots\}$. Then $\nexists$ any consecutive a,b s.t $\gcd(a,b)=1$

$|A|=4$

$\{1,2,3,4\}$

$B=3+1+1=5$.

$|B|=n+1$

Thus, $B=\{1,2,4,\ldots,2n\}$

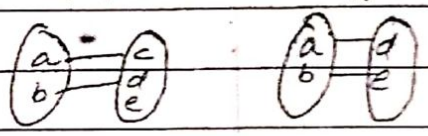
$\exists\, 1,2 \in B$ s.t $(1,2)=1$ and also 1,2 are consecutive.

$\Rightarrow |B|=n+1$. (d) ✓

1-1

$|A|=n$, $|B|=m$

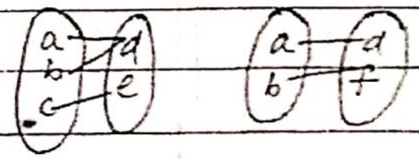
$\Rightarrow n \le m$



No. of 1-1 function $= n!\,{}^{m}C_{n}$

onto

$n \ge m$



No. of onto function = ?

Geometrical Def^n of 1-1 function

An function $f: A \to B$ is 1-1 if every line passing through B and parallel to A intersect the curve $y=f(x)$ at most once.

O.D DATE 1

Introduction to differential eq^n.

→ diff eq^n

→ class of diff eq^n

→ order of degree

→ Linear / Non Linear ODE

→ Sol^n of ODE

Net → $y' = y^{\alpha}$, $y(a)=0$, $\alpha \in (0,1)$

Chapter-2

Net JRF

Gate → First order first degree ODE → 3 Marks

→ Exact eq^n

→ Reducible into exact

→ Homogenius

→ Reducible into homogenius

Net → Linear eq^n

→ Reducible into Linear diff eq^n

Chapter-3

→ Higher order linear diff eq^n → 3 Que 2 Que → 4.75 Marks, 1 Que → 3 Marks

General Theory of Linear diff eq^n.

June # $|W|(y_1 \text{---} y_n)$

L.I / L.D Sol^ns

Dec # Zeros of any sol^n

$y'' + q(x)y = 0$

Sol^ns of 2^{nd} order L.D.E

Constant Coff of L.D.E

Chapter-4

4.75 → Uniqness and Existance of sol^n?

CH-5 → System of Linear equations.

CH-6 → Boundary value Problem

for example:- $\frac{d}{dx}\{x^2y'\} + 2xy' + \lambda y = 0$

$y(1) = 0, \quad y(10) = 0$

(a) $\exists\, \lambda_0$ such that $\forall\, \lambda \geq \lambda_0$, diff^n eq^n has non trivial solution.

(b) $A = \{\lambda \mid$ diff eq^n has non trivial sol$^n\}$ is dence in

(c)

(d) diff eq^n has two L.I sol^{n}s $\forall\, \lambda$ eigen value.

Chapter-1

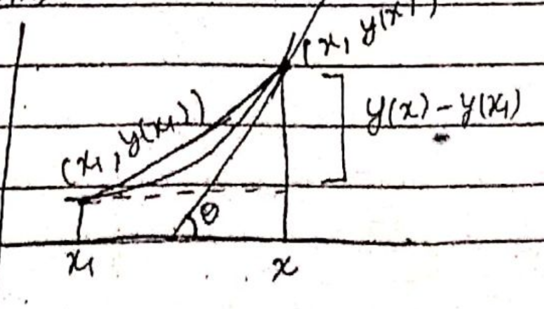
dependent / Independent variable

The variables whose value is assigned or domain is Known is called Independent variable and The variable whose value is obtained corresponding to assigned value is called dependent variable.

i.e. If f is a function $f: A \to B$ be a function
$\forall\, x \in A\ \exists$ unique $y \in B$ s.t $y = f(x)$
where y is called dependent variable, x is Independent variable.

Total ~~diff~~ derivative

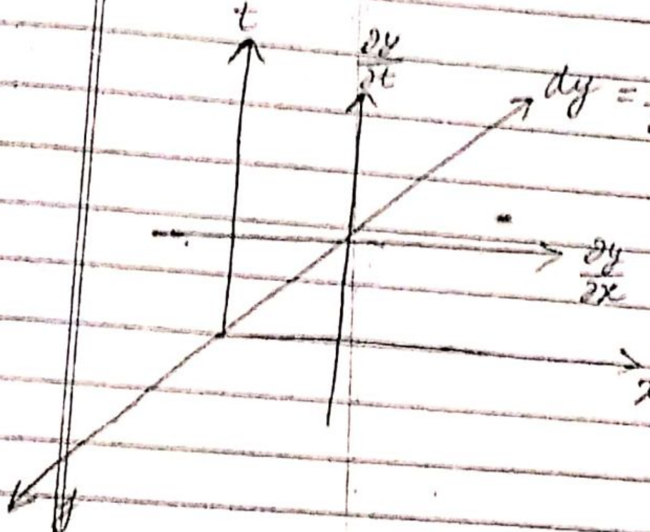
Let $y = y(x)$



$\tan\theta = \lim_{x_1 \to x} Slope(l) = \frac{y(x)-y(x_1)}{x-x_1} = \frac{\Delta y}{\Delta x}$ = Slope of tangent line at $= \frac{d}{dx}$

Note: To define derivative slope of tangent at point on the curve must be unique.

$y = y(x,t)$



<u>Total derivative</u>

The Total derivative is the rate of change in dependent variable w.r.t all of the Independent variables.

<u>Partial derivative</u>

The Partial derivative is rate of change in dependent variable w.r.t some of the Independent variables keeping others are fixed.

<u>Differential eq^n's</u>

If $x_1, x_2, \ldots, x_n$ are Independent variables and $y_1, y_2, \ldots, y_m$ are dependent variables.

Then the diff eq^n is an eq^n b/w dependent variables y_i

and the independent variable x and the derivatives.

Example :- 1. If $y = y(x)$

Then diff^n eq^n is $\frac{dy}{dx} + \frac{d^2y}{dx^2} = x$ is simple ordinary Diff^n eq^n

2. $y_1 = y_1(x)$, $y_2 = y_2(x)$

Then diff^n eq^n is
$$\left.\begin{aligned} \frac{dy_1}{dx} - \frac{dy_2}{dx} &= 0 \\ x\frac{dy_1}{dx} + \frac{dy_2}{dx} &= 0 \end{aligned}\right\} \text{ system of ODE}$$

3. $y = y(x,t)$

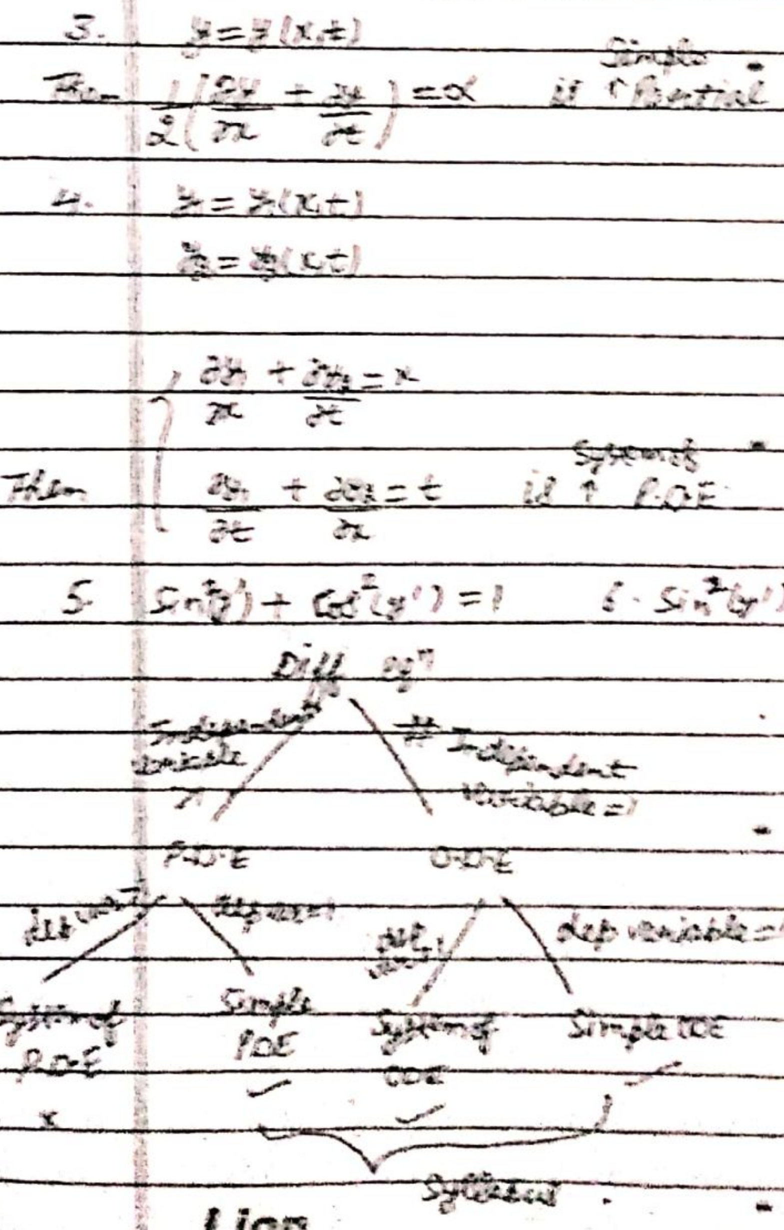
Then $\frac{1}{\alpha}\left(\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t}\right) = \alpha$ is simple Partial DE

4. $y_1 = y_1(x,t)$

$y_2 = y_2(x,t)$

Then $$\left\{\begin{aligned} \frac{\partial y_1}{\partial x} + \frac{\partial y_2}{\partial t} &= x \\ \frac{\partial y_1}{\partial t} + \frac{\partial y_2}{\partial x} &= t \end{aligned}\right. \text{ is system of PDE}$$

5. $\sin^2(y') + \cos^2(y') = 1$ $\sin^2(y') + \cos^2(y') = 2$



Lion

PD.

①

Definition of PDE - An equation containing one or more partial derivative of an unknown funⁿ of two or more independent variable is known as P.D.E.

Ex - $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1 \Rightarrow$ order 1, degree 1

$\frac{\partial z}{\partial x} \to p$, $\frac{\partial z}{\partial y} \to q$

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 1$ — order-2, degree-1

$\frac{\partial^2 z}{\partial x^2} \to r$, $\frac{\partial^2 z}{\partial x \partial y} \to s$, $\frac{\partial^2 z}{\partial y^2} \to t$

Order - order of PDE is defined as the Highest partial derivative occuring the PDE.

Degree - The degree of PDE is the power of highest order PDE which occur in it after the eqⁿ has been rationlized.

"Classification of Iˢᵗ order P.D.E"

1. **Linear PDE** - If first order P.D.E is said to be linear if the dependent variable and its partial derivative occur only in Iˢᵗ degree and the dependent variable & its partial derivative are not multiple of each other.

form of L.P.D.E - $P(x,y)\,p + Q(x,y)\,q = R(x,y)\,z + S(x,y)$

Ex
$$\left.\begin{aligned} yq &= 1 \\ px &= z \\ qy &= 1 \end{aligned}\right\} \text{L.P.D.E}$$

2. Semi Linear - If F.O.P.D.E is said to semi-Linear, it is Linear in p & q but not necessary in z and its partial derivative are not multiple of each other.

In the form

$$\boxed{P(x,y)p + Q(x,y)q = R(x,y,z)}$$

Ex

$$px + qy = z^2$$
$$qy = z^3$$
$$px^2 + y^2 q = z^2 - 1$$

3. Quassi Linear - A F.O.P.D.E is said to be quassi L.D.E if it is Linear in p & q.

$$P(x,y,z) + Q(x,y,z)q = R(x,y,z)$$

Ex

$$zp + xq = 1$$
$$p + z^2 q = 1 - z^2.$$

$$\boxed{L \Rightarrow S.L \Rightarrow Q.L}$$

Non-Linear PDE A F.O.P.D.E, which not satisfy above three form (F.OP.D.E) not is above 3 form)

Ex -

$$pq = 1$$
$$p^2 + q^2 = 1$$

Formation of P.D.E -

1. Elimination of arbitrary constant -

Consider an eq^n $F(x,y,z,a,b) = 0$ — (1)
where a & b denote arbitrary constant & let

(B)

z be regarded as a function of two independent variable of (x, y).

Diff w.r.t x & y partially of (1)

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z}\left(\frac{\partial z}{\partial x}\right) = 0 \quad -(2)$$

$(\because z = z(x, y))$

$$\left(\frac{\partial f}{\partial y}\right) + \left(\frac{\partial f}{\partial z}\right)p = 0 \quad -(2)$$

$$\left(\frac{\partial f}{\partial y}\right) + \left(\frac{\partial f}{\partial z}\right)q = 0 \quad -(3)$$

Eliminate two constant a & b from eq^n (1), (2) & (3) we shall obtain in

$$f(x, y, z, p, q) = 0$$

Que The partial D.E elliminating the constant $z = ax + by + ab$ is

(a) $z = px + qy - pq$

(b) $z = px + qy + pq$

(c) $z = px + qy - p^2q^2$

(d) $z = px + qy + p^2q^2$

$$\frac{\partial z}{\partial x} = a = p$$

$$\frac{\partial z}{\partial y} = b = q$$

Que $z^2(1+a^3) = 8(x+ay+b)^3$ then P.D.E is — (1)

(a) $p^3 + q^3 = 27z$

(b) $p^3 - q^3 = 27z$

(c) $p^2 + q^2 = 27z$

(d) $p^2 - q^2 = 27z$

$$2z\frac{\partial z}{\partial x}(1+a^3) = 24(x+ay+b)^2 \quad -(2)$$

$$2z\frac{\partial z}{\partial y}(1+a^2) = 24a(x+ay+b)^2$$

$$p/q = 1/a \quad -(3)$$

$(1)^2/(2)^3$

$\Rightarrow p^3(1+q^3) = 27z$

$p^3\left(1+q^3/p^3\right) = 27z$

$$\boxed{p^3 + q^3 = 27z}$$

Case Ist – When the No. of arbitrary constant C_N are equal to No. of independent variable I_N then we will get unique P.D.E of order 1

Case IInd If No. of arbitrary constant C_N is less than the No. of independent variable. then we ~~can~~ will have more than 1 independent variable get usually more than PDE of order one.

Ex – $z = ax + y$

$\frac{\partial z}{\partial x} = a = p$

$\frac{\partial z}{\partial y} = 1 \Rightarrow z = px + y$

$$\boxed{q = 1}$$

(If $C_N > I_N$; then we get more than one PDE. Of order greater than 1.)

Ex – $\frac{x^2}{a^2} + y^2/b^2 + z^2/c^2 = 1$

$z^2 = -c^2\left(x^2/a^2 + y^2/a^2\right)$ – (1)

$2z\frac{\partial z}{\partial x} = -c^2/a^2 \, 2x$

$\Rightarrow c^2 x + a^2 z p = 0$ – (2)

$2z\frac{\partial z}{\partial y} = -\frac{c^2}{b^2} 2y$

$c^2 = -a^2 z/x \, p$

S.K. Gupta

Real Analysis

Chap-01

→ functions, 1-1 functions, onto functions, Bijections
→ Similar Sets
→ Countable Sets → 1 Que
→ Uncountable sets

Chapter-02 (Point set Topology)

→ Bounded / Unbounded Sets
→ Sub/ Inf
→ Limit Point, Closed sets
→ Interior Point, open sets
→ Connected Sets } ****
→ Compact Sets
*** → Sum of Sets
→ Boundary Points

Chapter-03

→ Seq of Real no.s.
→ Bounded seq
→ Unbded seq
→ Limit Point of seq
→ Limit of seq
→ Monotonic seq
→ Subspace

2 Que (6-9 Marks)

→ Convergence of sequence → Practice

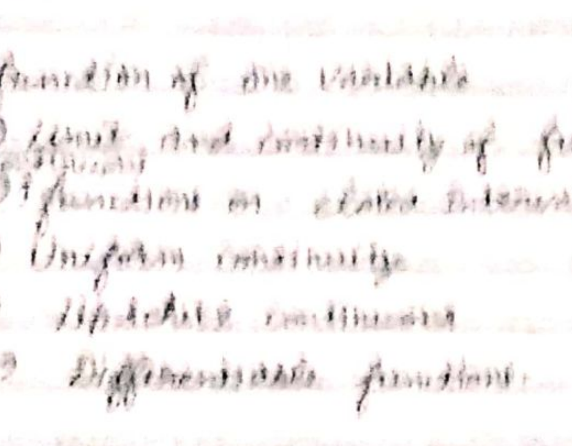
* Series of real numbers
* And series * Ideas
* Series of positive terms
* Alternating series
* Convergence of series

Chapter 04

* Function of one variable
* Limit and continuity of functions
* Functions on closed interval
* Uniform continuity
* Lipschitz continuous
* Differentiable functions

Ques: $d(x) = dist(x, A) = \inf\{|x-y| \mid y \in A\}$ $A = \{0,1\}$

$d(x) = \inf\{|x|, |x-1|\}$



$d(x)$ is not diff at 3 points

If A has n elements then $d(x)$ is not diff at $(2n-1)$ points

Chapter-05

Sequence / Series of functions → $\boxed{2\text{-}3 \text{ Que}}$

**** Chapter-06

Functions of Several variables → $\boxed{3\text{-}4 \text{ Que}}$
$f : \mathbb{R}^n \to \mathbb{R}^m$.

Chapter-01

Countable / Uncountable Sets

→ If A be any set:
Then $P(A) = \{ B \mid B \subseteq A \}$

→ If $|A| = n$, $|P(A)| = 2^n$
→ $\phi \in P(A) \Rightarrow P(A) \neq \phi$

→ If A is Infinite ⇒ P(A) is Infinite.

→ Function

Let A and B are any two nonempty ~~two~~ sets. Then $f: A \rightarrow B$ is called function if $\forall x \in A$ $\exists$ unique $y \in B$ such that $y = f(x)$.

→ If $|A| = n$, $|B| = m$
Then No. of functions from A to B $= m^n = |B|^{|A|}$

→ One-one function

Let $f: A \rightarrow B$ is a function. Then f is called 1-1 if
$\forall x, y \in A$, $x \neq y \Rightarrow f(x) \neq f(y)$
OR
$f(x) = f(y)$
$\Rightarrow x = y$

→ If $f: A \rightarrow B$ which is 1-1 function ⇒ $|A| \leq |B|$.
→ If $|A| > |B|$ ⇒ $\nexists$ any 1-1 function from A to B.

Complex Analysis

Unit- I

01 → complex number and function of complex variable.

02 → Limit, continuity diffⁿ. & C-R eqⁿ & Analytic fⁿ.

→ Training sheet. (1)

Unit-2

03 → Singularities

4 complex integration.

→ Training sheet. (2)

Unit-3

→ 15 marks.

05 – Liouville th. Picard th., Gauss & Luca's.

06 – Taylor Series, Identity th. and open mapping th.

→ Training sheet. (3)

→ training sheet (4).

Unit-IV 07- Power Series [cont. wise imp]

08- Laurent series and Residue

→ training sheet - 05.

~~Unit~~

Unit-V 09- extension of Liouville's th. maxima & minima modulo prin and Rungels th ~~principle~~.

10 – meromorphic & Rational fⁿ argument th., Roses th.

11 – Schwarz's lemma, Schwarz pic bilinear transformation and confor mation

→ t. s - 6

Standard Notation:-

$\mathbb{C} = \{x+iy \mid x, y \in \mathbb{R}\}$

$Re(z) =$ Real part of $z = x$.
$Im(z) =$ imaginary part of $z = y$

$\Delta =$ open unit disk $= \{z \in \mathbb{C} \mid |z| < 1\}$

$\bar{\Delta} = \{z \in \mathbb{C} \mid |z| \leq 1\}$.

$H(D) =$ set of holomorphic f^n or analytic f^n

$\mathbb{C}_\infty = \mathbb{C} \cup \{\infty\}$

$\bar{z} = x - iy$

$|z| = \sqrt{x^2+y^2}$

$|-z| = |z|$

$|\bar{z}| = |z|$

$\boxed{|z|^2 = z\bar{z}}$

$Re(z) = \dfrac{z+\bar{z}}{2} = x(z,\bar{z})$.

$Im(z) = \dfrac{z-\bar{z}}{2i} = y(z,\bar{z})$

$\overline{z_1 \pm z_2} = \bar{z_1} \pm \bar{z_2}$

$\overline{z_1 \cdot z_2} = \bar{z_1} \cdot \bar{z_2}$

$\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z_1}}{\bar{z_2}} \quad z_2 \neq 0.$

$|z_1 z_2| = |z_1| \cdot |z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1| + |z_2|$

$z = x + iy \sim (x, y) \in \mathbb{R}^2$

Polar form $z = re^{i\theta}$

$e^{i\theta} = \cos\theta + i\sin\theta$

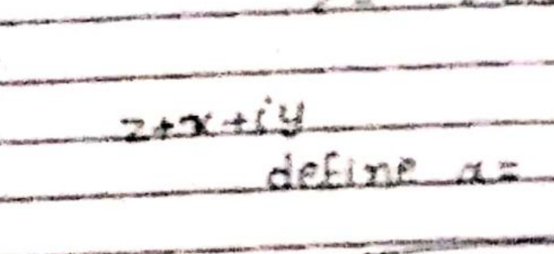
$z = r(\cos\theta + i\sin\theta)$

$\Rightarrow z = x + iy$

define $\alpha = \tan^{-1}\left|\frac{y}{x}\right|$, $\alpha \neq 0$

$\boxed{0 \leq \alpha < \pi/2}$

$$\text{Arg}(z) = \begin{cases} 0 & x > 0 \quad y = 0 \\ \alpha & x > 0 , y > 0 \\ \pi/2 & y > 0 \quad x = 0 \\ \pi - \alpha & y > 0 , x < 0 \\ \pm\pi & y = 0 \quad x < 0 \\ -\pi + \alpha & y < 0 \quad x < 0 \\ -\pi/2 & y < 0 , x = 0 \\ -\alpha & y < 0 , x > 0 \end{cases}$$



ALL MATERIAL AVAILABLE HERE

Hand Written Class Notes

JAM, GATE, NET for CSIR

MATHS, CHY, PHY, LIFE SCI .

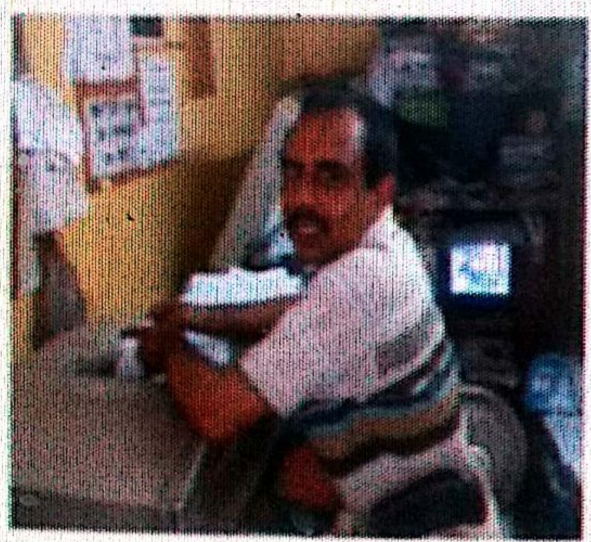
NET for UGC

ENG , ECO , HIS , GEO , PSCY , COM ENV,.... Etc.

GATE , IES , PSUs for ENGG.

ME, EC, EE, CS, CE .

IAS , JEE , NEET(PMT).



चौधरी PHOTO STAT

JIA SARAI NEAR IIT

DELHI - 110016

CONTACT NO: 9818909565

** All INDIA post also available**